

# A supercurrent switch in graphene $\pi$ -junctions

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We study the supercurrent in a superconductor/ferromagnet/superconductor graphene junction. In contrast to its metallic counterpart, the oscillating critical current in our setup decays only weakly upon increasing exchange field and junction width. We find an unusually large residual value of the supercurrent at the oscillatory cusps due to a strong deviation from a sinusoidal current-phase relationship. Our findings suggest a very efficient device for dissipationless supercurrent switching.

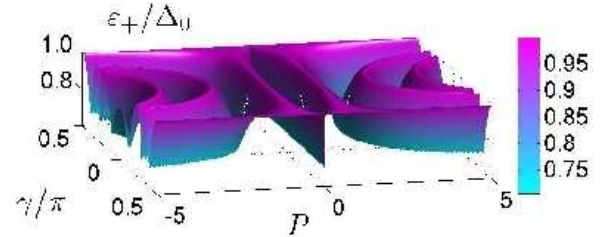
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Graphene is a condensed matter system displaying an emergent low-energy 'relativistic' electronic structure. For undoped graphene, the Fermi level reduces to six points, giving rise to nodal fermions at the edges of the Brillouin zone. Currently, it is of considerable interest, and potentially of great technological importance, to investigate how such unusual low-energy electronic structures manifest themselves in heterostructures where proximity effects are prominent. In particular, potential for future applications in devices seems plausible if such proximity-structures would combine two major functionalities in materials science, namely magnetism and superconductivity.

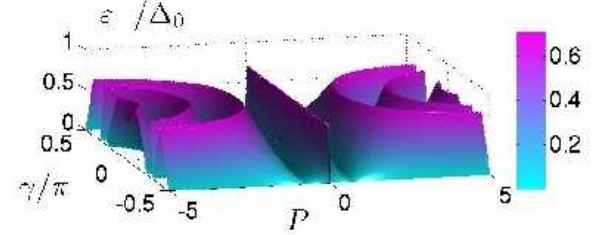
Recently, there have been several experimental reports on proximity-induced *superconductivity* in graphene [1, 2]. A measurable supercurrent was observed between regions of graphene under the influence of proximity-induced superconductivity in these works. Due to the massless nature and energy-independent velocity of the charge-carriers, graphene offers a unique environment for the manifestation of a Josephson effect. Unusual behavior for the supercurrent in a superconductor/normal/superconductor (S/N/S) graphene setup has been predicted, including an anomalous scaling behavior with the length of the normal region in the undoped case [3] and an oscillatory behavior as a function of gate voltage in the normal region [4].

Interestingly, it has also been shown [5, 6] that *ferromagnetic* correlations may be induced in graphene nanoribbons by means of external electrical fields. A suggestion for a more conventional magnetic proximity effect has also been put forth [7, 8], by means of exploiting a magnetic gate in contact with a graphene layer. The accompanying exchange splitting between the spin- $\uparrow$  and spin- $\downarrow$  electrons in graphene has been estimated [7] to lie around 5 meV for the magnetic insulator EuO. Precise estimates of the proximity induced exchange splitting are difficult in this case, due to the strong effect of the proximity layer on the magnetization in EuO [9]. Nevertheless, it is known that the magnetization in the proximity EuO layer is tunable [10]. In applications, this is a great advantage, since it in principle offers the possibility of a tunable proximity induced magnetization in graphene. Moreover, recent experiments on spin injection in a graphene layer show a rather long spin relaxation length  $\sim 1 \mu\text{m}$  at room temperature. This indicates that graphene is a promising material for

spin transport [7, 11].



a) The Andreev bound state  $\varepsilon_+$  [Eq. (3)] for  $\Delta\phi = \pi/2$ .



b) The Andreev bound state  $\varepsilon_-$  [Eq. (3)] for  $\Delta\phi = \pi/2$ .

FIG. 1: (Color online) Contour-plot of the Andreev bound states in the ferromagnetic region carrying the current between the superconductors.

The following question arises naturally: Do novel physical effects arise due to the peculiar electronic properties of graphene and simultaneously the interplay between ferromagnetic and superconducting correlations? The wide range of exotic phenomena that originate from the mutual interplay between magnetic and superconducting order include  $0-\pi$  transitions [12, 13], odd-frequency pairing [14], and even intrinsic coexistence of ferromagnetism and superconductivity in the same material [15, 16]. In particular, from the viewpoint of applications, the possibility of altering the fundamental Josephson current-phase relationship in a controlled fashion may bring about potential implications for their use in superconducting electronics as well as in (quantum) logic circuits based on superconductors [17].

In this Letter, we investigate the interplay between proximity-induced superconductivity and ferromagnetism in a graphene layer, resulting in an unusual behavior of the supercurrent through the system. Our main results are: *i*) The current-phase relationship deviates strongly from sinusoidal

behavior, indicating a significant contribution from higher harmonics and *ii*) the critical current at the  $0-\pi$  transition is finite and has a much larger value than the one observed in metallic systems. The latter result suggests a very efficient performance of the device as a supercurrent switch.

We envisage an experimental setup where superconductivity is induced in two parts of the graphene region by means of conventional superconductors, such as Nb or Al, in close proximity. Between the superconducting regions, an exchange splitting is induced in the graphene layer by means of *e.g.* a magnetic insulating material. Instead of using a magnetic insulator such as EuO, where one in principle could tune the magnetization in the proximity layer with an external magnetic field, one also could envision using a multiferroic (*e.g.* BiFeO<sub>3</sub>) or piezomagnetic material (*e.g.* Fe<sub>x</sub>Ni<sub>y</sub>B<sub>z</sub>) in close proximity to the graphene layer. Both of these classes of materials would offer the opportunity of tuning the exchange field in the material by some external control parameter – electric field due to the magnetoelectric coupling in the former case, and pressure in the latter. Upon modifying the exchange field in the proximity layer of the material, it is reasonable to assume that the proximity-induced exchange field in graphene would also be altered. Materials in which the magnetoelectric coupling is substantial are currently attracting much interest due to their potential for novel technological applications [18]. In order to control the local Fermi level in the ferromagnetic (F) region, one could possibly use a normal gate on top of the magnetic insulator to create a tunable barrier [7]. The superconducting (S) regions are assumed to be heavily doped, such that the Fermi energy satisfies  $\varepsilon_F \gg \Delta$ , while the F region is taken to be undoped, *i.e.*  $\varepsilon'_F \simeq 0$ . Moreover, we assume sharp edges for the region separating the F and S graphene regions, and focus on the short-junction regime which is experimentally feasible.

We will proceed to show that the Josephson current in an S/F/S graphene junction displays a strong oscillatory, non-monotonic dependence on both the exchange field  $h$  and width  $d$  of the junction. Most interestingly, we find a large residual value of the supercurrent at the cusps of these oscillations. This indicates a sign reversal of the current, and the considerable residual value of the supercurrent at these cusps suggests a very efficient device for dissipationless supercurrent switching. We now present our results in detail.

The F region separating the superconductors is taken to be undoped, such that the effective Fermi level is  $\sigma h$  for spin- $\sigma$  electrons. The regions S must be strongly doped to justify the mean-field treatment of superconductivity. We assume that this is comparable to the estimated exchange-splitting in the F region [7, 8]. Thus, we take  $\varepsilon_F \simeq h$  to obtain analytically tractable results. To construct the scattering states that carry the supercurrent across the F region, we write down the Bogoliubov-de Gennes equations [19] in the presence of an exchange field  $h$ . The Bogoliubov-de Gennes equation essentially describes the eigenstates of quasiparticles in each of the graphene regions and their belonging eigenvalues  $\varepsilon$ . It may be obtained by diagonalizing the full Hamiltonian, and con-

stitutes the foundation for constructing the scattering states which are involved in the transport formalism we here use. For the spin-species  $\sigma$ , one finds that

$$\begin{pmatrix} H_0 - \sigma h(x) & \sigma \Delta(x) \\ \sigma \Delta^*(x) & -H_0 - \sigma h(x) \end{pmatrix} \begin{pmatrix} u^\sigma \\ v^{-\sigma} \end{pmatrix} = \varepsilon \begin{pmatrix} u^\sigma \\ v^{-\sigma} \end{pmatrix}. \quad (1)$$

Here, we have made use of the valley degeneracy and defined  $H_0 = v_F \mathbf{p} \cdot \boldsymbol{\sigma}$ , where  $\mathbf{p}$  is the momentum vector in the graphene plane and  $\boldsymbol{\sigma}$  is a vector of Pauli matrices. The superconducting order parameter  $\Delta(x)$  couples electron- and hole-excitations in the two valleys located at the two inequivalent corners of the hexagonal Brillouin zone. The  $u^\sigma$  spinor describes the electron-like part of the total wavefunction  $\psi^\sigma = (u^\sigma, v^{-\sigma})^T$ , and in this case reads  $u^\sigma = (\psi_{A,+}^\sigma, \psi_{B,+}^\sigma)^T$  while  $v^{-\sigma} = \mathcal{T}u^\sigma$ . Here,  $^T$  denotes the transpose while  $\mathcal{T}$  is the time-reversal operator. To capture the essential physics, we write  $\Delta(x) = \Delta_0 e^{i\phi_{L,R}}$  in the left and right S region and  $\Delta(x) = 0$  otherwise. Similarly, we set  $h(x) = h$  in the F region and  $h = 0$  otherwise. The Josephson current is computed via the usual energy-current relation summed over projections of all paths perpendicular to the tunneling barrier [20]

$$I_J(\Delta\phi) = -\frac{2e}{\hbar} \sum_i \int_{-\pi/2}^{\pi/2} \frac{d\gamma \cos \gamma}{f^{-1}[\varepsilon_i(\Delta\phi)]} \frac{d\varepsilon_i(\Delta\phi)}{d\Delta\phi}, \quad (2)$$

where  $\varepsilon_i(\Delta\phi)$  are the Andreev bound states carrying the current in the F region, and  $\Delta\phi = \phi_R - \phi_L$  is the macroscopic phase difference between the superconductors. The integration over angles  $\gamma$  takes into account all possible trajectories and  $f(x)$  is the Fermi-Dirac distribution function. We define the critical supercurrent as  $I_c = |\max\{I_J(\Delta\phi)\}|$  and introduce  $I_0 = 2e\Delta_0$ . The procedure for obtaining  $\varepsilon_i(\Delta\phi)$  is the same as in Ref. [3] and the details will be given elsewhere; here we give the main results. By introducing  $T(\gamma, P) = 4 \sin^2 \gamma \sin^2(P \cos \gamma) + \cos^4 \gamma$  and  $\Phi(\gamma, P) = [2 \sin^2(P \cos \gamma) - \cos^2 \gamma] \cos^2 \gamma \cos \Delta\phi - \cos^4 \gamma - 4 \sin^2 \gamma \sin^2(P \cos \gamma)$ , we find that the allowed bound states have energies  $\pm \varepsilon_\sigma(\Delta\phi)$  ( $\sigma = \pm$ ) with

$$\varepsilon_\sigma(\Delta\phi) = \frac{\Delta_0}{\sqrt{2T(\gamma, P)}} \left[ \sigma \{ \Phi(\gamma, P)^2 - 4T(\gamma, P) [\cos^2 \gamma \times \cos^2(\Delta\phi/2) - \sin^2(P \cos \gamma)]^2 \}^{1/2} - \Phi(\gamma, P) \right]^{1/2}. \quad (3)$$

The parameter  $P = hd/v_F$  captures the effect of both the exchange field  $h$  and the length  $d$  of the junction. To understand the nature of these bound states, consider Fig. 1 for a representative plot of  $\varepsilon_\pm(\Delta\phi)$ , using  $\Delta\phi = \pi/2$ . As is seen from both plots, the bound state energies exhibit a strong oscillatory dependence on the parameter  $P$ . This indicates that similar oscillatory behavior may be expected in the supercurrent itself. Interestingly, the oscillations seen in Fig. 1 are not damped with increasing  $P$ . This directly reflects the Dirac-cone linear dispersion of the graphene electrons and is reminiscent of the weak damping of conductance oscillations at subgap energies in graphene-superconductor junctions [21, 22].

Inserting the derivative of Eq. (3) into Eq. (2) provides the supercurrent. The current-phase relationship for the S/F/S graphene junction is shown in Fig. 2. With increasing  $P$ , the

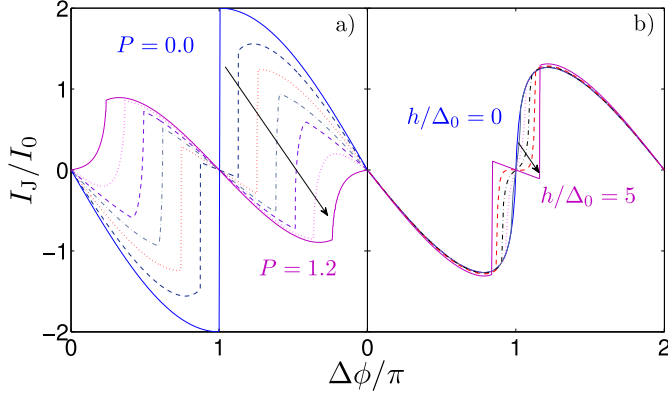


FIG. 2: (Color online) a) Current-phase relationship in the S/F/S graphene junction with undoped F region. We have fixed  $\varepsilon_F = h$  and set  $\varepsilon'_F = 0$ . We have used values of  $P$  in the interval  $[0.0, 1.2]$  in steps of 0.2. b) Current-phase relationship in the S/F/S graphene junction with doped F region. We have set  $\varepsilon_F/\Delta_0 = 10$ ,  $\varepsilon'_F/\Delta_0 = 15$ ,  $d/\xi = 0.05$  and vary  $h/\Delta_0$  in the range  $[0, 5]$  in steps of 1.

critical current gets suppressed and finally the sign of the current is changed. Remarkably, the critical current never goes to zero. An interesting feature of the plot in Fig. 2a) is that the discontinuity at  $\Delta\phi = \pi$  for  $P = 0$  is split for increasing  $P$ . The discontinuity of the current-phase relation originates with a crossing of Andreev levels in the normal graphene (F graphene with  $h = 0$ ) region at  $\Delta\phi = \pi$ . For  $\Delta\phi \in [0, \pi)$ , only the 0-mode Andreev bound state carries the current. For  $\Delta\phi \in (\pi, 2\pi]$ , the  $\pi$ -mode Andreev bound state carries the current, such that there is an abrupt crossover exactly at  $\Delta\phi = \pi$ . The situation changes when  $h \neq 0$ , since the spin-splitting doubles the number of Andreev bound states. Consequently, the crossover between different modes may occur at  $\Delta\phi \neq \pi$ , as a result of the superharmonic current-phase relationship. We have checked explicitly that the strong deviation from a sinusoidal current-phase relationship persists for larger  $d$  that do not satisfy  $d/\xi \ll 1$ . However, in this case one should strictly speaking also include the contribution to the current from the continuum of supergap states [20]. This requires a separate study, and we here focus on the short-junction regime.

To show that the splitting of this discontinuity originates with the presence of an exchange field which separates the spin- $\uparrow$  and spin- $\downarrow$  bands, we have also numerically solved the current-phase relationship for a nonzero Fermi level in the ferromagnetic region. Although we have obtained analytical results in this regime, these are somewhat cumbersome and therefore omitted here. The result is shown in Fig. 2b) where we have chosen  $\Delta_0 = 1$  meV,  $\varepsilon_F = 10$  meV, and  $\varepsilon'_F = 15$  meV, and varying  $h$  in the range  $[0, 5]$  meV. This ensures that there are no evanescent modes, such that only the Andreev bound states carry the current. We choose a junction with

$d/\xi = 0.05$ , where  $\xi$  is the superconducting coherence length, since the short junction regime  $d \ll \xi$  is the experimentally most relevant one. The figures in a) and b) correspond to two quite different regimes: in a) the exchange field is much larger than the Fermi level while in b) the exchange field is much smaller than the Fermi level. The trend is nevertheless seen to be the same in both cases, namely a progressive splitting of the discontinuity located at  $\Delta\phi = \pi$  in the paramagnetic case.

Assuming a heavily doped superconducting region with  $\varepsilon_F = 10$  meV and an effective gap  $\Delta_0$  of 1 meV, a mean-field treatment is justified by  $\varepsilon_F \gg \Delta_0$ . Moreover, the short-junction regime requires that  $d \ll \xi$ . Using  $v_F \simeq 10^6$  m/s in graphene, we obtain from  $\xi = v_F/\Delta$  that  $d \ll 650$  nm is required. This condition has been met in at least two ex-

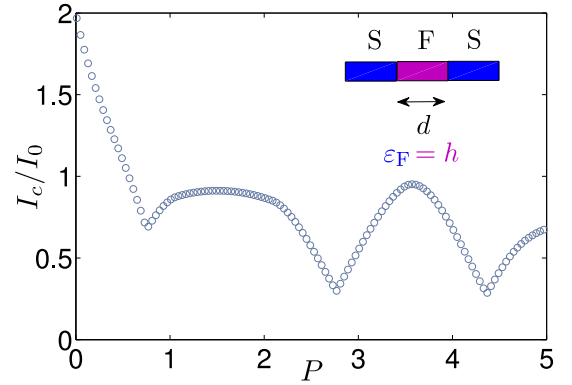


FIG. 3: (Color online) The critical supercurrent in a proximity-induced S/F/S graphene junction for  $\varepsilon_F = h$  and  $\varepsilon'_F = 0$ .

perimental studies of proximity-induced superconductivity in graphene [1, 2]. The critical supercurrent  $I_c$  for an S/F/S graphene junction for  $\varepsilon_F = h$  and  $\varepsilon'_F = 0$  is shown in Fig. 3. The critical current shows oscillations with respect to  $P$ , but decays weakly compared to the metallic case and never reaches  $I_c = 0$  in the relevant regime. For instance, there is a factor  $\simeq 100$  in reduction of the amplitude of the current right after the second cusp in the metallic case for  $h \simeq 10\Delta_0$  (see Fig. 2 in Ref. 23) while there is only a factor  $\simeq 2$  in reduction of the amplitude in the present case. Right at the cusps located at  $P \simeq \{0.8, 2.8, 4.4\}$ , there is a large residual value of the supercurrent which should be experimentally detectable. This is very distinct from the usual sinusoidal current-phase relationship for the Josephson current, in which the supercurrent vanishes completely at the  $0-\pi$  transition. The first switch occurs at a value  $P = hd/v_F \approx 0.8$ . For an exchange splitting of  $h \simeq 10$  meV, this requires a junction width  $d = 50$  nm. Alternatively, employing a junction width of  $d = 100$  nm [1, 2] one would need an exchange splitting of  $h \simeq 5$  meV [7].

In order to explain the appearance of cusps in the critical current dependent on exchange field and junction width, it is instructive to draw parallels to the metallic S/F/S junction and the behavior of the supercurrent. In most experimental situations, the effective barriers separating the F and S regions

are strong, leading to a current-phase relationship which is very nearly sinusoidal, *i.e.*  $I_c = I_0 \sin \Delta\phi$  [24]. By tuning the temperature  $T$  and width of the junction  $d$ , one is able to switch the sign of the amplitude  $I_0$ , which necessarily means that one must have  $I_0 = 0$  at some point. Precisely such behavior has been observed in several experiments [12, 25]. In the present system, the current-phase relationship deviates strongly from sinusoidal behavior, and contains a significant contribution from higher harmonics. Tracking the absolute value of the current with increasing  $P$  from Fig. 2, it is seen that  $I_c$  never becomes zero. Instead, it has a large residual value at the points where the current changes sign. While a small, but finite value of the supercurrent at the  $0-\pi$  transition also has been observed in metallic S/F/S junctions [26], the magnitude of the residual value of the supercurrent in the graphene case is huge compared to the metallic case.

Since we have assumed a homogeneous chemical potential in each of the S and F graphene regions, the experimental observation of the predicted effects require charge homogeneity of the graphene samples. This is a challenge, since electron-hole puddles in graphene imaged by a scanning single electron transistor device [27] suggest that such charge inhomogeneities play an important role in limiting the transport characteristics of graphene close to the Dirac point [28]. In doped graphene, as considered here, we expect that the inhomogeneities should play a smaller role than in undoped graphene. Although we have neglected the spatial variation of the superconducting gap near the S/F interfaces, we do not expect our qualitative results to be affected by taking into account the reduction of the gap. Also, we have assumed that the junction  $d$  is short enough to neglect the orbital effect the magnetic field constitutes on the electrons.

In summary, we have investigated the interplay between proximity-induced superconductivity and ferromagnetism in a graphene layer. In contrast to its metallic counterpart, the oscillating supercurrent in our setup decays only weakly upon increasing exchange field and junction width. We find huge residual values of the supercurrent at the  $0-\pi$  transition points where the supercurrent changes sign. If the exchange splitting could be adjusted by means of some external source, such as gate voltage or external electrical fields [5, 6, 18], these results imply that the supercurrent across the junction may be tuned in a controllable fashion. Specifically, a very efficient

supercurrent switch may be realized by tuning the exchange field infinitesimally near the sign reversal points.

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